

# Withdrawal Location with Progressive Tax Rates

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*Optimal withdrawal strategies are developed for retirees with multiple types of tax-advantaged savings accounts. In an environment of progressive tax rates, the ability to convert pretax funds in traditional IRAs at low tax rates substantially increases investors' residual accumulations and withdrawal sustainability. Specifically, informed withdrawal-location strategies, in which traditional IRA distributions can be applied against exemptions, deductions, and lightly taxed tax brackets, can increase residual accumulations by more than \$1 million. In these strategies, the optimal tax bracket through which an investor should take distributions is directly related to the investor's wealth level.*

**A**s a growing percentage of the population enters retirement, increasing numbers of investors are facing critical decisions about withdrawing funds from tax-advantaged savings accounts, sometimes referred to as tax-deferred savings accounts (TDAs). Moreover, as defined-contribution plans become increasingly popular, retirees and those planning for retirement are forced to make decisions that past generations did not have to make. The introduction of various tax-advantaged savings accounts [e.g., Roth IRAs, Section 529 plans, and Roth 401(k) plans] complicates matters and increases the decisions retirees face.

Fortunately, researchers have recognized these trends and developed models to help guide financial planners and investors. Most advances thus far, however, have focused on tax-efficient investment decisions. That is, research has examined the optimal choice among various TDAs (e.g., those with front-end tax benefits versus those with back-end tax benefits), Roth IRA conversion decisions, and early-withdrawal penalties.<sup>1</sup> In *Tax-Advantaged Savings Accounts and Tax-Efficient Wealth Accumulation* (Horan 2005b), my intention was to provide a review and synthesis of the literature on tax-efficient investing and extend or advance previous technologies in a cohesive theoretical framework.

Tax-efficient withdrawal, the focus of this article, has attracted less attention. Retirees with multiple types of tax-advantaged accounts from which to withdraw, however, would benefit from guidance regarding optimal withdrawal policies. For exam-

ple, should traditional IRA or Roth IRA balances be drawn down first? Should a retiree use some combination of withdrawals from both accounts?

The term "withdrawal location" derives from the literature on asset location, which analyzes the types of accounts (taxable or tax advantaged) in which investors should hold bonds and equity.<sup>2</sup> In the 1980s, Black (1980) and Tepper (1981) proffered an arbitrage argument for corporate pension funds that can be extended to individual investors to suggest that they should place highly taxed assets, such as bonds, in tax-sheltered accounts and place tax-preferenced securities, such as equity, in taxable accounts. To the extent that bonds are undesirable to the investor from an asset allocation perspective, he or she should offset lending in the tax-sheltered account with borrowing in the taxable account. Other authors have extended this basic premise to consider such factors as borrowing constraints, the use of municipal bonds, trading behavior, and liquidity constraints.<sup>3</sup>

Several authors have investigated the sustainability of retirement withdrawals, but few have focused on tax-efficient withdrawal policies.<sup>4</sup> An exception is work by Ragsdale, Seila, and Little (1993, 1994), who approached the issue from the perspective of tax codes. They developed a mathematical programming model that incorporated a myriad of then-prevailing tax regulations governing retirement distributions, including early-withdrawal penalties, minimum distribution taxes, excess distribution taxes, and estate taxes. Since the publication of their work, the Taxpayer Relief Act of 1997 (TRA) has repealed excess distribution taxes and introduced the Roth IRA, changing the calculus and balance of considerations. Also, in an unpublished manuscript (Horan 2005a), I showed

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heuristically that if tax rates vary over time, retirees with multiple types of TDAs can benefit from taking traditional IRA distributions when tax rates are low and taking Roth IRA distributions when tax rates are high. I developed that work in a deterministic framework, however, that lacks the insights that Monte Carlo analysis would provide.

In this article, in contrast to those works, I develop a model that focuses on the salient features of a tax code with multiple marginal tax rates and abstracts from the code's seemingly infinite loopholes and minutiae. Specifically, the model recognizes that withdrawals from some types of accounts are taxed as ordinary income and withdrawals from other accounts are not taxed. Therefore, the ability to choose the type of account from which to make withdrawals is valuable and the model is designed to highlight opportunities for tax-efficient withdrawal locations created by a progressive tax rate system.

The analysis considers a progressive tax rate structure characterized by exemptions and deductions. This approach is meaningful for investors for whom retirement account withdrawals represent the totality of taxable income. In that case, withdrawals are subject to a series of progressively higher tax rates. This study also has implications for the literature addressing after-tax valuation of assets held in TDAs.<sup>5</sup> Although the models vary in their degree of sophistication, none recognizes that a portion of traditional IRA distributions may be tax exempt or subject to a series of marginal tax rates. Therefore, this study may help advance our understanding of after-tax valuation of TDAs.

## The Model

The withdrawal model is designed for the presence of a progressive tax rate scheme with six tax brackets that allow exemptions and deductions. Withdrawals from a tax-advantaged account with front end-loaded tax benefits, such as a traditional IRA, are taxed as ordinary income. In contrast, withdrawals from tax-advantaged accounts, such as a Roth IRA, are not taxed.<sup>6</sup> It applies to investors who have little to no exogenous taxable income that might otherwise deplete the exemptions, deductions, or tax brackets with modest rates—that is, in cases where retirement withdrawals represent the totality of taxable income.<sup>7</sup> And the model is designed to analyze the impact of various withdrawal algorithms on residual accumulations and withdrawal sustainability.

Consider a retiree with balances in both a traditional IRA and a Roth IRA, the pretax balance in each

account at any time  $t$  is represented by, respectively,  $V_{Trad,t}$  and  $V_{Roth,t}$ . The portfolios are similarly invested to generate a pretax expected return,  $r$ , over some withdrawal horizon,  $n$ . The retiree intends to make after-tax withdrawals initially equal to  $w$  percent of the combined pretax value of the two accounts at the beginning of the year. The retiree increases this initial after-tax withdrawal annually by  $g$  to maintain purchasing power. Therefore,

$$W_t \equiv w(V_{Trad,0} + V_{Roth,0})(1+g)^{t-1}$$

represents the retiree's after-tax withdrawal requirement for period  $t$ .

The retiree can implement one of eight withdrawal strategies: two naive strategies and six informed strategies. The first naive strategy satisfies the retiree's after-tax withdrawal requirement by making withdrawals from the traditional IRA each year until its balance is depleted, at which time, withdrawals from the Roth IRA commence. The second naive strategy reverses this sequence by satisfying the retiree's after-tax withdrawal requirement from the Roth IRA until its balance is depleted and then from the traditional IRA.

The six informed distribution strategies withdraw from the traditional IRA balance, if available, either up to the exemption and deduction limit or up to a specified tax bracket. The available exemptions and deductions and the level of tax brackets grow by the rate of inflation,  $\pi$ , over time.<sup>8</sup> Any additional funds required to satisfy the after-tax withdrawal requirement are distributed from the Roth IRA.

For example, consider a retiree with a \$50,000 after-tax withdrawal requirement and with exemptions and deductions totaling \$16,400. In the tax regime, taxable income up to \$14,600 is taxed at 10 percent. Based on the first of the informed withdrawal strategies, the retiree would take \$16,400 from the traditional IRA and the remaining \$33,600 from the Roth IRA, in which case the investor would pay no tax on distributions. In the second informed withdrawal strategy, the retiree would take \$31,000 from the traditional IRA. Of this amount, \$16,400 would escape taxation and \$14,600 would be taxed modestly at 10 percent, generating a total after-tax distribution of \$29,540. The remaining \$20,460 after-tax required distribution would be withdrawn from the Roth IRA. The remaining informed withdrawal strategies would be similarly constructed except that each withdrawal from the traditional IRA would increase taxable distributions to the next tax bracket. Appendix A contains mathematical algorithms for these withdrawal strategies.

## Residual Accumulations and Withdrawal Sustainability

Scenario analysis can help decipher the effects of withdrawal strategies on residual accumulations and withdrawal sustainability. The base case is a retiree with \$1,000,000 in pretax funds in a traditional IRA and \$666,667 in a Roth IRA. These balances are comparable on a taxable-equivalent basis because distributions from these two accounts are taxed differently.<sup>9</sup> The base case also assumes that a retiree has a 6 percent after-tax withdrawal requirement, so the initial after-tax distribution is \$100,000 [that is,  $0.06 \times (\$1,000,000 + \$666,667)$ ], and this distribution grows by 3 percent annually (i.e.,  $g = 3$  percent) over a 25-year withdrawal horizon.<sup>10</sup> For a meaningful comparison of residual accumulations of traditional and Roth IRAs at the end of this time period, I compare them on a taxable-equivalent basis, similar to the view of the initial balances.<sup>11</sup> For example, the “traditional, then Roth” withdrawal strategy might leave \$80,000 remaining in the Roth IRA at the end of 25 years. At an 8 percent return, this sum would produce annual after-tax withdrawals of \$11,922 for 10 years. The residual accumulation would then be converted to a “taxable equivalent”—that is, a sum held in a taxable account that would produce the same after-tax 10-year annuity produced by the Roth IRA. I apply a similar procedure, assuming a 28 percent tax rate on distributions, to residual balances in the traditional IRA.<sup>12</sup> In any case, the reported results are insensitive to these conditions.

The analysis begins with the personal exemptions, standard deduction, and tax brackets for a married couple filing jointly in 2005, as displayed in **Table 1**. Of course, individual circumstances vary, but the analysis can be modified accordingly.<sup>13</sup> The levels of the exemptions and tax brackets generally increase over time by the rate of inflation. The base case assumes that these values increase at the 2.5 percent inflation rate. A pretax return of 8 percent is also assumed.

Panel A of **Table 2** displays residual accumulations 25 years hence and withdrawal sustainability under various after-tax withdrawal rates for each of the six strategies: the two naive strategies (withdrawing from either the traditional IRA or Roth IRA until the balance is depleted, then withdrawing from the other) and the six informed strategies (taking distributions from the traditional IRA up to the available exemptions and deductions or up to the top of a specified tax bracket and satisfying the remainder of the withdrawal requirement from the Roth IRA). According to Panel A, withdrawing from the traditional IRA until funds are exhausted

**Table 1. Data for a Married Couple Filing Jointly in 2005**

Adjusted Gross Income		Taxed at Rate
From	To	
\$ 0	\$ 16,400 <sup>a</sup>	0%
16,400	31,000 <sup>b</sup>	10
31,000	75,800	15
75,800	136,350	25
136,350	199,200	28
199,200	342,850	33
342,850	Greater	35

<sup>a</sup>This amount represents personal exemptions plus the standard deduction. Taxpayers over the age of 65 are entitled to an additional standard deduction of \$1,000 per spouse.

<sup>b</sup>This amount represents the top of the 15 percent tax bracket plus the personal exemptions and deductions, or \$14,600 plus \$16,400. The remaining amounts in this column should be interpreted analogously.

before withdrawing from the Roth IRA (Naive 1) is the better of the two naive strategies. Taking initial distributions from the traditional IRA rather than the Roth IRA is preferable because a portion of the distribution is applied to exemptions and deductions and a portion is taxed very lightly—at 10 percent or 15 percent. In fact, distributions up to \$75,800 are taxed at 15 percent or less. This modest tax burden decreases the pretax distribution required to generate the after-tax withdrawal requirement, leaving more after-tax assets in tax-sheltered accounts. Panel B shows that incremental residual accumulations from Naive 1 over Naive 2 range from about \$150,000 to about \$900,000.

The traditional, then Roth IRA strategy (Naive 1) performs far better than the reverse strategy because Naive 2 wastes chances in early retirement years to take traditional IRA distributions that are shielded from tax. Although the Naive 2 strategy avoids taxes in the initial years of retirement, it misses opportunities to convert pretax funds in traditional IRAs to after-tax funds at “low” rates (as the reader will see, “low” is a relative term). Note that neither of the naive withdrawal strategies sustains withdrawal rates greater than 6 percent for the 25-year horizon.

The performance of the informed strategies in comparison with Naive 1 is presented in Panel B of Table 2. The informed withdrawal strategies take distributions from the traditional IRA equal to the allowable exemptions and deductions or up to the top of a specific tax bracket. The remainder of the withdrawal requirement is satisfied from the Roth IRA. According to Panel B, only one informed strategy performs substantially better than Naive 1. Taking distributions from the traditional IRA

**Table 2. Residual Accumulations and Withdrawal Sustainability for Various Strategies and Withdrawal Rates**

Strategy	Withdrawal Rate, $w$								
	4.0%	4.5%	5.0%	5.5%	6.0%	6.5%	7.0%	7.5%	8.0%
<i>A. Residual accumulations and withdrawal sustainability</i>									
Naive 1: Trad., then Roth	4,612,749	3,648,393	2,704,153	1,772,786	869,832	[24]	[21]	[19]	[17]
Naive 2: Roth, then trad.	3,735,164	3,000,054	2,250,068	1,491,069	722,091	[24]	[21]	[18]	[16]
Withdrawal from trad.									
Up to exemption	3,878,717	3,142,400	2,389,500	1,622,084	846,444	61,375	[21]	[19]	[17]
Up to 10% bracket	4,087,292	3,237,748	2,468,517	1,697,960	921,667	131,395	[22]	[19]	[17]
Up to 15% bracket	4,649,409	3,799,865	2,950,321	2,100,777	1,251,233	401,689	[23]	[20]	[18]
Up to 25% bracket	4,612,749	3,648,393	2,704,153	1,772,786	869,832	[24]	[21]	[19]	[17]
Up to 28% bracket	4,612,749	3,648,393	2,704,153	1,772,786	869,832	[24]	[21]	[19]	[17]
Up to 33% bracket	4,612,749	3,648,393	2,704,153	1,772,786	869,832	[24]	[21]	[19]	[17]
<i>B. Incremental residual accumulations and withdrawal sustainability over Naive 1</i>									
Naive 2: Roth, then trad.	-877,585	-648,339	-454,085	-281,717	-147,741	[0]	[0]	[1]	[1]
Withdrawal from trad.									
Up to exemption	-734,033	-505,993	-314,653	-150,702	-23,388	nmf	[0]	[0]	[0]
Up to 10% bracket	-525,457	-410,644	-235,635	-74,826	51,835	nmf	[1]	[0]	[0]
Up to 15% bracket	36,660	151,472	246,168	327,991	381,400	nmf	[2]	[1]	[1]
Up to 25% bracket	0	0	0	0	0	[0]	[0]	[0]	[0]
Up to 28% bracket	0	0	0	0	0	[0]	[0]	[0]	[0]
Up to 33% bracket	0	0	0	0	0	[0]	[0]	[0]	[0]

Notes: For an investor with \$1 million in a traditional IRA and \$666,667 in a Roth IRA. The amounts in brackets represent the number of years over which withdrawals are fully sustained.

nmf = not a meaningful figure.

through the 15 percent tax bracket with any additional withdrawal requirement met from Roth IRA distributions is optimal. This strategy can produce residual accumulations almost \$400,000, or more than 40 percent, greater than Naive 1. And the difference tends to increase, although not monotonically, with the withdrawal rate. At high withdrawal rates, the optimal withdrawal strategy produces an extra year or two of sustainability. Making traditional IRA withdrawals through the 25 percent or higher tax brackets produces identical results to the Naive 1 strategy because it yields identical withdrawal patterns; that is, distributions are taken from the traditional IRA until funds are depleted. As the reader will see, the optimal withdrawal strategy for wealthier investors is to make withdrawals through higher tax brackets.

Note that, despite the assumption of stable returns, no informed strategy can sustain growing withdrawals greater than 6.5 percent of the initial pretax values beyond 25 years. Part of the reason is that the base case assumes that the nominal value of the withdrawal is growing at a higher rate than the value of the tax brackets. Furthermore, sustainable withdrawal rates in a dynamic environment in which returns are a random variable that varies from one year to the next are usually

lower than they are in a stable, deterministic environment (see Milevsky and Robinson 2005).

Among the sources of uncertainty in this analysis is the long-term rate of inflation. Therefore, it is useful to know whether inflation rates, which drive the nominal growth in tax brackets, affect the relative value of the various withdrawal strategies. **Table 3** presents outcomes of the strategies for various long-term inflation rates and tells a consistent story. Table 3 assumes the base-case withdrawal rate of 6 percent and assumes that this withdrawal grows at a nominal rate of 3 percent regardless of the rate of inflation. Although withdrawing from the traditional IRA first (Naive 1) is the better naive strategy, making withdrawals through the 15 percent tax bracket produces the best outcomes, regardless of the inflation rate. Other informed strategies also perform better than Naive 1 in some situations. Specifically, at low inflation rates, making withdrawals through the top of the 25 percent tax bracket performs better than Naive 1. Making withdrawals through the 15 percent tax bracket, however, is still optimal. At high inflation rates, making withdrawals through the 10 percent bracket performs better than Naive 1. Again, differences approach \$400,000.

**Table 3. Residual Accumulations and Withdrawal Sustainability for Various Withdrawal Strategies by Inflation Rate**

Strategy	Inflation Rate, $\pi$								
	1.0%	1.5%	2.0%	2.5%	3.0%	3.5%	4.0%	4.5%	5.0%
<i>A. Residual accumulations and withdrawal sustainability</i>									
Naive 1	819,581	837,613	853,836	869,832	886,292	903,228	920,655	938,588	957,042
Naive 2	589,019	633,216	679,324	722,091	760,729	802,432	847,449	896,050	944,147
Withdrawal from trad.									
Up to exemption	720,025	761,250	805,280	846,444	883,378	923,271	965,795	1,011,133	1,055,811
Up to 10% bracket	795,754	833,140	877,461	921,667	959,914	995,232	1,033,484	1,081,723	1,119,973
Up to 15% bracket	1,098,153	1,147,432	1,199,995	1,251,233	1,251,643	1,253,376	1,250,943	1,252,000	1,251,194
Up to 25% bracket	830,620	840,199	853,836	869,832	886,292	903,228	920,655	938,588	957,042
Up to 28% bracket	819,581	837,613	853,836	869,832	886,292	903,228	920,655	938,588	957,042
Up to 33% bracket	819,581	837,613	853,836	869,832	886,292	903,228	920,655	938,588	957,042
<i>B. Incremental residual accumulations and withdrawal sustainability over Naive 1</i>									
Naive 2	-230,562	-204,397	-174,512	-147,741	-125,563	-100,796	-73,206	-42,538	-12,895
Withdrawal from trad.									
Up to exemption	-99,556	-76,363	-48,556	-23,388	-2,914	20,042	45,139	72,545	98,770
Up to 10% bracket	-23,827	-4,473	23,625	51,835	73,622	92,004	112,828	143,135	162,931
Up to 15% bracket	278,571	309,819	346,159	381,400	365,351	350,148	330,288	313,412	294,152
Up to 25% bracket	11,039	2,586	0	0	0	0	0	0	0
Up to 28% bracket	0	0	0	0	0	0	0	0	0
Up to 33% bracket	0	0	0	0	0	0	0	0	0

Notes: For an investor with \$1 million in a traditional IRA and \$666,667 in a Roth IRA. Withdrawal rate = 6 percent; withdrawal growth rate = nominal 3 percent.

Panel A of **Table 4** displays the incremental outcomes of the best informed strategy (making withdrawals from the traditional IRA through the 15 percent tax bracket) over Naive 1 for various withdrawal tax rates and pretax returns. The advantage of the informed strategy generally increases with the pretax return and the withdrawal rate. Specifically, the incremental value of the informed strategy is greatest for high returns and high withdrawal rates, with incremental accumulations exceeding \$500,000 in many cases. This result indicates that following an informed strategy is particularly important for retirees with aggressive investment and withdrawal strategies. Panel B indicates that the outperformance of the best informed strategy over the worst naive strategy, Naive 2, is enormous—with incremental residual accumulations of approximately \$2 million in some cases.

Consider now an investor who is twice as wealthy as the one in this base case; that is, this higher-net-worth (HNW) investor has \$2 million in a traditional IRA and \$1.33 million in a Roth IRA. For this investor, Panel A of **Table 5** shows that the optimal strategy is to take taxable withdrawals from the traditional IRA through the 25 percent tax bracket—an informed strategy that more than doubles the residual accumulation of either naive strategy. Panel B indicates that the incremental

accumulations can be almost \$800,000 more than those of Naive 1. As previously, informed withdrawal-location strategies can add an additional year or two to withdrawal sustainability. Panel C examines for the HNW investor the incremental accumulation for various pretax rates of return. At high rates of return and aggressive withdrawal rates, incremental residual accumulation over Naive 1 approaches \$1.5 million. Comparing these results with those in **Table 4** shows that these differences can be two to three times as large as the differences for the more moderately endowed investor. In unreported results, I found that incremental residual accumulations can exceed \$2 million.

**Table 6** displays incremental accumulations for an investor endowed with \$3 million of traditional IRA assets and \$2 million of Roth IRA assets (a HNW+ investor). Panel A indicates that the optimal strategy for this investor is to take taxable distributions from the traditional IRA up through the top of the 28 percent tax bracket, in which case residual accumulations can be about \$1.2 million greater than Naive 1 provides. Alternatively, the investor can sustain withdrawals for an additional year or two. At higher rates of withdrawal and pretax return, the investor's residual accumulations may be more than \$2 million greater than Naive 1 provides (see Panel B).

**Table 4. Incremental Accumulations and Withdrawal Sustainability of Best Informed Strategy vs. Naive Strategies**

Pretax Return, $r$	Withdrawal Rate, $w$								
	4.0%	5.0%	6.0%	7.0%	8.0%	9.0%	10.0%	11.0%	12.0%
<i>A. Best informed strategy compared with Naive 1</i>									
4.0%	11,007	13,120	19,359	25,860	36,660	-9,801	-10,651	-11,590	-12,629
5.0%	[1]	[1]	154,942	196,209	246,168	186,960	75,441	-63,952	-132,543
6.0%	[1]	[1]	[1]	nmf	381,400	350,836	290,313	209,975	76,204
7.0%	[0]	[1]	[1]	[1]	[2]	492,904	461,616	427,419	367,318
8.0%	[0]	[0]	[0]	[1]	[1]	[1]	[2]	658,583	581,295
9.0%	[1]	[0]	[0]	[0]	[0]	[1]	[1]	[2]	842,716
10.0%	[0]	[1]	[0]	[0]	[0]	[0]	[1]	[1]	[1]
<i>B. Best informed strategy compared with Naive 2</i>									
4.0%	93,007	217,295	391,243	616,064	914,245	1,166,237	1,472,935	1,854,903	2,276,311
5.0%	[1]	[1]	246,447	442,633	700,253	901,968	1,151,434	1,463,537	1,856,915
6.0%	[1]	[1]	[1]	134,353	529,142	711,215	899,745	1,165,882	1,497,068
7.0%	[0]	[1]	[1]	[1]	[2]	549,371	725,433	951,426	1,207,116
8.0%	[0]	[0]	[0]	[1]	[2]	[1]	[2]	806,279	1,030,683
9.0%	[1]	[0]	[0]	[1]	[0]	[1]	[1]	[2]	930,775
10.0%	[0]	[1]	[0]	[0]	[0]	[0]	[1]	[2]	[1]

Notes: For an investor with \$1 million in a traditional IRA and \$666,667 in a Roth IRA. The "best informed strategy" is defined as making withdrawals from the traditional IRA through the 15 percent tax bracket. Figures in brackets represent the incremental number of years over which withdrawals are fully sustained.

nmf = not a meaningful figure.

In unreported results, I found that taking taxable distributions from the traditional IRA up to the top of the 28 percent tax bracket is also optimal for investors with \$4 million in traditional IRA assets. For investors with a \$6 million balance in a traditional IRA, this informed strategy remains optimal in many situations, but taking taxable distributions from the traditional IRA up through the top of the 33 percent tax bracket may be preferable for higher withdrawal rates. Part of the reason that taking taxable distributions from the traditional IRA up through the 28 percent tax bracket remains optimal in many situations is that the increment to the next tax bracket represents 5 percentage points (33 percent minus 28 percent) rather than 3 percentage points (28 percent minus 25 percent).

## Conclusion

This study investigated the effects of various withdrawal patterns on residual accumulations and withdrawal sustainability for retirees having two types of tax-advantaged savings accounts from which to make withdrawals in a progressive tax rate regime. Because distributions from traditional IRAs are taxable and those from Roth IRAs are not, making withdrawals from traditional IRAs is advantageous when the tax burden is light and making withdrawals from Roth IRAs is advanta-

geous when the tax burden would otherwise be heavy. When taxable distributions can be applied against personal exemptions and deductions or against tax brackets with relatively low tax rates, a naive strategy of withdrawing from the traditional IRA first was found to perform substantially better than the naive strategy of withdrawing from the Roth IRA first. However, an informed strategy of taking traditional IRA distributions up to the top of a "low" tax bracket and satisfying the remainder of the withdrawal requirement from the Roth IRA yields residual accumulations that are substantially greater than the better naive strategy.

Of course, minimum distribution requirements from traditional IRAs dictate to some extent required withdrawals from traditional IRAs after age 70 1/2. To the extent retirees have discretion over accounts from which they make withdrawals, however, they can manage their tax obligations and dramatically affect their residual accumulations and withdrawal sustainability.

Light and heavy tax burdens mean different things to different investors. The definition of a "low" tax bracket is directly related to an investor's wealth. I found that for retirees with initial balances in traditional and Roth IRAs of, respectively, \$1 million and \$667,000, the optimal strategy is to take taxable distributions through the top

**Table 5. Residual Accumulations and Withdrawal Sustainability for the HNW Investor**

*A. Residual accumulations and withdrawal sustainability*

Strategy	Withdrawal Rate, <i>w</i>								
	4.0%	4.5%	5.0%	5.5%	6.0%	6.5%	7.0%	7.5%	8.0%
Naive 1	8,053,069	6,192,727	4,332,948	2,468,964	626,853	[22]	[19]	[17]	[15]
Naive 2	7,172,878	5,576,834	3,919,076	2,241,291	535,219	[22]	[19]	[17]	[15]
Withdrawal from trad.									
Up to exemption	7,398,864	5,789,938	4,123,259	2,432,715	718,471	[22]	[19]	[17]	[15]
Up to 10% bracket	7,535,905	5,917,392	4,242,863	2,542,995	833,386	[23]	[20]	[17]	[16]
Up to 15% bracket	8,038,681	6,339,593	4,643,161	2,922,385	1,180,053	[23]	[20]	[18]	[16]
Up to 25% bracket	8,215,521	6,516,433	4,817,345	3,118,257	1,419,169	[24]	[20]	[18]	[16]
Up to 28% bracket	8,053,069	6,198,733	4,488,889	2,789,801	1,090,713	[23]	[20]	[18]	[16]
Up to 33% bracket	8,053,069	6,192,727	4,332,948	2,468,964	626,853	[22]	[19]	[17]	[15]

*B. Incremental residual accumulations and withdrawal sustainability over Naive 1*

Strategy	Withdrawal Rate, <i>w</i>								
	4.0%	4.5%	5.0%	5.5%	6.0%	6.5%	7.0%	7.5%	8.0%
Naive 2	-880,191	-615,892	-413,872	-227,673	-91,635	[0]	[0]	[0]	[0]
Withdrawal from trad.									
Up to exemption	-654,205	-402,789	-209,689	-36,249	91,618	[0]	[0]	[0]	[0]
Up to 10% bracket	-517,164	-275,334	-90,084	74,031	206,533	[1]	[1]	[0]	[1]
Up to 15% bracket	-14,388	146,866	310,213	453,421	553,200	[1]	[1]	[1]	[1]
Up to 25% bracket	162,453	323,707	484,397	649,293	792,316	[2]	[1]	[1]	[1]
Up to 28% bracket	0	6,006	155,942	320,837	463,860	[1]	[1]	[1]	[1]
Up to 33% bracket	0	0	0	0	0	[0]	[0]	[0]	[0]

*C. Best informed strategy compared with Naive 1 strategy*

Pretax return, <i>r</i>	Withdrawal Rate, <i>w</i>								
	4.0%	5.0%	6.0%	7.0%	8.0%	9.0%	10.0%	11.0%	12.0%
4.0%	111,358	139,499	198,438	236,182	162,453	59,832	0	0	0
5.0%	[1]	[1]	397,233	492,688	484,397	468,184	415,752	352,797	249,674
6.0%	[0]	[1]	[1]	[1]	792,316	853,783	890,125	936,720	1,000,058
7.0%	[0]	[1]	[1]	[1]	[1]	[1]	1,174,363	1,343,135	1,474,675
8.0%	[0]	[0]	[0]	[1]	[1]	[1]	[1]	nmf	1,664,095
9.0%	[0]	[1]	[0]	[0]	[0]	[1]	[0]	[1]	[2]
10.0%	[0]	[0]	[0]	[1]	[0]	[0]	[0]	[1]	[1]

Notes: For an investor with \$2 million in a traditional IRA and \$1.33 million in a Roth IRA. The “best informed strategy” is defined as making withdrawals from the traditional IRA through the 25 percent tax bracket. Figures in brackets represent the number of years over which withdrawals are fully sustained.

nmf = not a meaningful figure.

of the 15 percent tax bracket. Residual accumulations may be \$400,000, or 40 percent, greater than the better naive strategy (Naive 1). For initial balances in traditional and Roth IRAs of, respectively, \$2 million and \$1.33 million, the optimal strategy was found to be making taxable withdrawals through the top of the 25 percent tax bracket. Residual balances in this case could be \$800,000 to \$1.5 million for investors with high-return portfolios and aggressive withdrawal strategies. When initial traditional and Roth IRAs balances are, respectively, \$3 million and \$2 million, I found that the optimal strategy is to make withdrawals

through the top of the 28 percent tax bracket. Incremental accumulations were found to exceed \$1 million and could exceed \$2.5 million for investors with high-return portfolios and aggressive withdrawal strategies.

In general, the value of an informed strategy increases as the aggressiveness of both the retiree’s investment strategy and withdrawal requirements increases. As a result, retirees can significantly improve the sustainability of their retirement portfolios by embarking on an optimal withdrawal program that exploits opportunities to take traditional IRA distributions at low tax rates.

**Table 6. Incremental Accumulations and Withdrawal Sustainability for the HNW+ Investor***A. Incremental residual accumulations and withdrawal sustainability over Naive 1*

Strategy	Withdrawal Rate, $w$								
	4.0%	4.5%	5.0%	5.5%	6.0%	6.5%	7.0%	7.5%	8.0%
Roth then Trad.	-619,178	-381,985	-243,143	-109,156	-90,339	[0]	[0]	[0]	[0]
Trad. to exemption	-336,603	-117,865	3,820	106,523	95,455	[0]	[1]	[1]	[0]
Trad. to 10% bracket	-157,137	30,193	162,344	233,105	261,719	[0]	[1]	[1]	[0]
Trad. to 15% bracket	351,061	532,091	608,983	681,158	647,166	[1]	[1]	[1]	[0]
Trad. to 25% bracket	527,902	747,442	927,666	994,119	941,822	[1]	[1]	[1]	[0]
Trad. to 28% bracket	527,902	747,442	927,666	1,142,672	1,235,763	[2]	[2]	[1]	[1]
Trad. to 33% bracket	0	0	13,529	227,463	362,451	[1]	[1]	[1]	[0]

*B. Best informed strategy compared with Naive 1 strategy*

Pretax Return, $r$	Withdrawal Rate, $w$									
	4.0%	5.0%	6.0%	7.0%	8.0%	9.0%	10.0%	11.0%	12.0%	
4.0%	262,441	336,910	439,532	532,912	527,902	500,983	479,161	560,472	645,717	
5.0%	[0]	[1]	nmf	842,905	927,666	1,026,338	1,162,437	1,264,916	1,396,507	
6.0%	[1]	[0]	[1]	[1]	1,235,763	1,491,103	1,718,239	1,958,873	2,268,839	
7.0%	[0]	[0]	[1]	[1]	[2]	[2]	nmf	2,396,025	2,868,060	
8.0%	[0]	[1]	[1]	[1]	[1]	[1]	[1]	323,389	2,622,180	
9.0%	[1]	[0]	[0]	[1]	[1]	[1]	[1]	[1]	[2]	
10.0%	[1]	[0]	[1]	[0]	[0]	[1]	[1]	[0]	[1]	

Notes: For an investor endowed with \$3 million of traditional IRA assets and \$2 million of Roth IRA assets. The “best informed strategy” is defined as making withdrawals from the traditional IRA through the 28 percent tax bracket. Figures in brackets represent the number of years over which withdrawals are fully sustained.

nmf = not a meaningful figure.

This theme generalizes well to other tax rate structures. For example, distributions from traditional IRAs may not be subject to multiple marginal tax rates but to a single, uniform marginal tax rate because other taxable income exhausts the opportunity to use exemptions, deductions, and tax brackets with low tax rates. Nonetheless, if tax rates vary over time (because of changes in political regimes, fiscal policy, or the investor’s tax status), the overriding lesson remains: Withdraw funds from traditional IRAs when funds will be taxed at low rates.

Several avenues for future research are suggested by this study. First, mandatory distribution requirements from traditional IRAs could be incorporated into the withdrawal models.

Second, this study suggests valuable withdrawal strategies to the extent that minimum distribution requirements allow. This constraint may not be binding in many circumstances because the optimal naive strategy calls for the retiree to deplete the traditional IRA before making Roth IRA withdrawals. Furthermore, the optimal informed strategy calls for significant withdrawals from the traditional IRA anyway. In either case, the retiree is making substantial traditional IRA distributions. Therefore, future research might formally incorporate the effect of traditional IRA distributions on the

taxation of Social Security benefits or implement similar withdrawal models that assume a different return structure. Third, future research might integrate the impact on withdrawal location of taxable accounts outside traditional and Roth IRA plans.

Finally, on a fundamental level, this study highlights the notion that investors derive option value from having more than one type of account from which to make retirement withdrawals. Therefore, a contingent-claims analysis assuming a stochastic marginal tax rate based on Monte Carlo simulations would help estimate the value to investors of having withdrawal flexibility. The insights from this analysis would then have implications for an investor’s initial asset-location decision. Alternatively, one could apply a Monte Carlo analysis to different tax rate regimes by using simulated returns based on historical time series of various asset classes, based on an assumed distribution with parameters derived from historical experience, or based on a factor model.

At any rate, the results contained in this article provide valuable lessons to retirees and their advisers. In particular, withdrawal strategies should capitalize on opportunities to convert pretax funds in traditional IRAs to after-tax funds at low tax rates.

*This article qualifies for 1 PD credit.*

## Appendix A. Algorithms for Withdrawal Strategies

To model the withdrawal strategies, the first step is to establish a function that defines the pretax distribution that would be required to generate a particular after-tax cash flow,  $X$ , in a progressive tax rate structure characterized by exemptions and deductions at low income levels and six tax brackets. Such a function,  $P$ , can be expressed as

$$P(X) = \min(X, A_{0,t}) + \sum_{j=1}^6 \frac{\max[0, \min(X, A_{j,t}) - A_{j-1,t}]}{1 - T_{bj}}, \quad (A1)$$

where  $A_{j,t}$  is the top of the  $j$ th tax bracket in period  $t$  and  $T_{bj}$  is the applicable tax rate for bracket  $j$ . In this formulation,  $A_{0,t}$  represents exemptions and deductions available in period  $t$ . So,  $A_{1,t}$  is the sum of  $A_{0,t}$  and the amount of income that could be taxed at  $T_{b1}$ .

Note that the tax brackets in Equation A1,  $A_{j,t}$ , are expressed on an after-tax basis. For example, for the retiree discussed in the text with a \$50,000 after-tax withdrawal requirement and with exemptions and deductions totaling \$16,400,  $A_1$  is not \$31,000, or \$16,400 plus \$14,600. Rather,  $A_1$  is \$29,540, the after-tax income resulting from \$31,000 of taxable income.

The inverse of this function, the after-tax proceeds of a pretax withdrawal,  $Y$ , can be expressed as

$$F(Y) = \min(Y, B_{0,t}) + \sum_{j=1}^6 \max[0, \min(Y, B_{j,t}) - B_{j-1,t}](1 - T_{bj}), \quad (A2)$$

where  $B_{j,t}$  is the top of the  $j$ th tax bracket expressed on a pretax basis in period  $t$ . For instance,  $B_0$  would be \$16,400 and  $B_1$  would be \$31,000 in the example.

**Naive Withdrawal Strategies.** For either IRA account type  $i$ , let the value of the account at time  $t$  be

$$V_{i,t} \equiv V_{i,t-1}(1 + r), \quad (A3)$$

which reflects the effect of the tax-sheltered investment earnings on asset growth. Therefore, the traditional IRA distribution for the first naive strategy, in which the retiree draws from the traditional IRA until funds are depleted, can be expressed as

$$D_{1,Trad,t} = \min[P(W_t), V_{Trad,t}]. \quad (A4)$$

The first value inside the minimum operator,  $P(W_t)$ , is the pretax withdrawal from the traditional IRA necessary to generate the after-tax withdrawal requirement,  $W_t$ . It recognizes that portions of the pretax withdrawal are taxed at potentially six dif-

ferent rates. The second value inside the minimum operator,  $V_{Trad,t}$ , is the remaining traditional IRA balance if it is insufficient to satisfy the after-tax withdrawal requirement.

The Roth IRA distribution is then

$$D_{1,Roth,t} = \max\{0, \min[W_t - F(D_{Trad,t}), V_{Roth,t}]\}, \quad (A5)$$

which implies that withdrawals from the Roth IRA should commence once the balance in the traditional IRA becomes insufficient to meet the after-tax withdrawal requirement. These withdrawals are constrained to not exceed the available Roth IRA balance. Notice that the pretax distribution function in Equation A1 is not applied to  $W_t$  because withdrawals from Roth IRAs are not taxed, whereas traditional IRA distributions are considered on an after-tax basis.

The residual balance in either type of IRA at withdrawal horizon  $n$  can now be expressed as the future value of the initial balance less the future value of any distributions:

$$V_{i,n} = V_0(1 + r)^n - \sum_{t=1}^n D_{i,t}(1 + r)^{n-t}. \quad (A6)$$

The second naive withdrawal strategy directs the retiree to make withdrawals from the Roth IRA as long as funds are available, followed by withdrawals from the traditional IRA. In this case, the Roth IRA withdrawal algorithm is

$$D_{2,Roth,t} = \min[W_t, V_{Roth,t}]. \quad (A7)$$

The traditional IRA withdrawal is similar to Equation A4, but the desired after-tax income from the traditional IRA is not  $W_t$ . It is the difference between the after-tax withdrawal requirement and the Roth IRA distribution, if positive. Substituting  $\max(0, W_t - D_{Roth,t})$  for  $W_t$  in Equation A4 yields

$$D_{2,Trad,t} = \min\left\{P\left[\max(0, W_t - D_{Roth,t})\right], V_{Trad,t}\right\}. \quad (A8)$$

**Informed Withdrawal Strategies.** Each of the six informed withdrawal strategies specifies withdrawals from the traditional IRA up to some exemption or tax bracket limit,  $B_{j,t}$ , to the extent that funds are available and to the extent that the withdrawal requirement demands the withdrawals. The first informed withdrawal strategy makes traditional IRA withdrawals up to allowable exemptions and deductions,  $B_{0,t}$ . The second informed withdrawal strategy makes traditional IRA withdrawals up to the first tax bracket,  $B_{1,t}$ , and so on. The remaining withdrawal requirement, if any, would be distributed from the Roth IRA. Each of these strategies for tax bracket  $j$  can be expressed with the following algorithms,

$$D_{j,Trad,t} = \max \left\{ \min \left[ B_{j,t}, V_{Trad,t}, P(W_t) \right], \min \left[ P(W_t - V_{Roth,t}), V_{Trad,t} \right] \right\} \quad (A9)$$

and

$$D_{j,Roth,t} = \min(W_t - D_{Trad,t}, V_{Roth,t}). \quad (A10)$$

The first algorithm, Equation A9, stipulates a traditional IRA withdrawal up to the top of the  $j$ th tax bracket in period  $t$ ,  $B_{j,t}$ , subject to fund availability and the size of the withdrawal requirement. The second algorithm, Equation A10, indicates that the remaining withdrawal requirement will be distributed from the Roth IRA to the extent that funds are

available. If the Roth IRA contains insufficient funds for the remainder of the withdrawal, the traditional IRA is tapped for the additional withdrawal, which is represented in the second value in the braces of Equation A9. One advantage of these strategies is that they relax, at least to some degree, the constraints imposed by traditional IRA minimum distribution requirements, which are not explicitly incorporated into the models. That is, minimum distribution requirements mandate traditional IRA withdrawals that these informed strategies call for anyway, potentially making the minimum distribution constraint nonbinding.

## Notes

- For example, Burgess and Madeo (1980); Bogan and Bogan (1982); O'Neil, Saftner, and Dillaway (1983); Mano and Burr (1984); Doyle (1984); Randolph (1994); Crain and Austin (1997); Horan, Peterson, and McLeod (1997); Krishnan and Lawrence (2001); Horan and Peterson (2001); Horan (2003, 2004); Gokhale and Kotlikoff (2003); Prakash and Smyser (2003); Benven (2003); Kitces (2003).
- I thank an anonymous referee for suggesting the use of the term "withdrawal location."
- Authors who have examined asset location include Auerbach and King (1983), Reichenstein (2001), Shoven and Sialm (2003), Huang (2003), and Dammon, Spatt, and Zhang (2004). A proper treatment of asset location requires an understanding of the after-tax value of tax-sheltered assets. Reichenstein (1998), Sibley (2002), Horan (2002), Reichenstein and Jennings (2003), and Poterba (2004) are among those who have addressed the issue of valuing tax-sheltered assets on an after-tax basis.
- Papers focusing on the sustainability of withdrawals include Cooley, Hubbard, and Walz (1998, 1999, 2001, 2003a, 2003b), Tezel (2004), Ervin, Filer, and Smolira (2005), and Milevsky and Robinson (2005). Sabelhaus (2000) analyzed aggregate accumulations and withdrawals for the United States.
- For example, Reichenstein and Jennings (2003); Poterba (2004); Horan (2002, 2005b).
- For succinctness, I use the term "traditional IRA" to represent generically all tax-deferred accounts with front-loaded tax benefits, including 401(k), 403(b), 457, and Keogh plans. Similarly, the term "Roth IRA" is understood as generally representing TDAs with back-loaded tax benefits, such as Section 529, Roth 401(k), and lifetime savings accounts.
- Therefore, the applicability of the model developed in the article is broader than the terminology I use might suggest.
- Alternatively, an investor might have exogenous income that subjects retirement distributions to a single marginal tax rate. I investigated this setting in Horan (2005a).
- Inflation rate  $\pi$ , by which tax brackets are assumed to grow over time, is distinct from  $g$ , the nominal growth rate of the retiree's withdrawal amount. The simulations assume that  $\pi$  is 2.5 percent and  $g$  is 3.0 percent, so the growth rate of the retiree's withdrawal does more than preserve purchasing power; that is, withdrawals grow slightly in real terms over time.
- Sibley (2002) and Horan (2002) showed that the taxable equivalent of a traditional IRA equals  $(1 - T_n)$  of a Roth IRA with the same pretax balance. Others have advocated after-tax approaches (e.g., Reichenstein 1998; Reichenstein and Jennings 2003), but the distinction is irrelevant in this case.
- Keep in mind that growth in nominal distributions is distinct from the general rate of inflation.
- Because a portion of Social Security benefits may, depending on the retiree's total income and marital status, be taxable, a retiree's marginal tax rate can be a bit elusive. Traditional IRA withdrawals are treated as income, so these distributions can make some Social Security benefits subject to tax, thereby increasing the effective marginal tax rate.
- I supply details of this conversion procedure in Horan (2005b). An alternate and more straightforward approach is to evaluate residual balances on an after-tax basis.
- In addition to the additional standard deduction of \$1,000 for a spouse that taxpayers over the age of 65 are entitled to, taxpayers may have itemized deductions that would shield additional distributions from taxation.

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